2017-01-06 Exam problems, Architecture {examiner: Dr. Adam Marlewski}

1. **Plane Cartesian (*Oxy*) and polar ($O\rho\theta$) coordinate systems; relations $x = \rho\cdot\cos\theta$, $x = \rho\cdot\sin\theta$.**
2. **Equations of plain curves given explicitly (e.g., $y = \text{atan}\, x$; $\rho = \exp\theta$ – a Bernoulli/logarithmic spiral), implicitly (e.g., $x^2/a^2 + y^2/b^2 = 1$ – an ellipse) and parametrically (e.g. $x = a\cdot(t - \sin t)$, $y = a\cdot(1 - \cos t)$ – a cycloid; $x = a\cdot(2\cos t - \cos 2t)$, $y = a\cdot(2\sin t - \sin 2t$ – a cardioid).**
3. **Plane vectors: the point, Euclid(ean) and Descartes/Cartesian definitions: (P, Q), (P, m, k, s), $(x_P, y_P; x_Q, y_Q)$. A free vector (and the equivalency relation in the set of all plane vectors).**
4. ***n*-dimensional vectors.**
5. **Dot/inner/scalar product of vectors: $u\cdot v$, or $<u, v>$, on the plane, in the space *R*3, in a space R^n.**
6. **Cross product of vectors: $u\times v$ (geometrical definition, analytical formula).**
7. **Equations of a line on the real plane (i.e., in the space R^2) and in R^3.**
8. **Equations of a plane (in R^3).**
9. **Matrices, their types (e.g., a square matrix, an upper triangular matrix, an invertible/non-singular matrix) and their algebra (the equality, the sum, the difference, the Cauchy product).**
10. **Determinant of the matrix – its definition, Laplace expansion, properties.**
11. **Sale (a system of algebraic linear equations), its solvability (Kronecker-Capelli theorem) and methods to solve them (via Cramer's rules, via inverse matrix, via Gauss elimination method).**
12. **Polynomial collocation problem – the Vandermonde technique, the Lagrange polynomial.**
13. **Polynomial least-square approximation/fit.**
14. **Number/numerical sequences (with examples, also that defined recursively, e.g. Fibonacci sequence), their types (a.o., monotone, increasing, bounded, divergent, convergent; with examples, e.g., arithmetic sequence, geometric sequence, 0-1 sequence). The necessary condition a sequence (a_n) to be convergent ($a_{n+1}-a_n \to 0$ as $n \to \infty$). Algebra of sequences and of their limits (their equality, sum, difference; the sandwich theorem). The theorem on a monotone and bounded sequence.**
15. **Bernoulli (or sequential) definition of the (Euler) number *e*. Graphs $y = e^{\lambda x}$ (with $\lambda > 0$, $\lambda = 0$ and $\lambda < 0$), $y = \cosh x$ – a catenary, $y = \sinh x$.**
16. **Number/numerical series: the definition, the convergence, the necessary condition a series Σa_k to be convergent ($a_k \to 0$ as $k \to \infty$), the quotient/d'Alembert and the root/Cauchy criteria of convergence, the Leibniz criterion (concerning an alternating series). Examples: harmonic ($1+1/2+1/3+1/4+\ldots=\infty$), Brounckner (aka alternating harmonic, $1-1/2+1/3-1/4+\ldots=\ln 2$), Huygens ($\Sigma 1/t_k=2$, where t_k is *k*-th triangle number), Madhava/Gregory/Leibniz ($1-1/3+1/5-1/7\ldots=\pi/4$), Basel ($1+1/2^2+1/3^2+1/4^2+\ldots=\pi^2/6$), Euler ($1+1/2!+1/3!+1/4!+\ldots=e$).**
17. **Function sequences (e.g., $1/x^n$, $x\in<0,1>$) and series (incl. Taylor series, e.g., for $1/(1-x)$, $\exp x$, $\sin x$).**
18. **Differential calculus: the definition of the derivative, its geometrical and physical interpretations. Algebra of differentiation (the derivative of a linear combination, of the product and of the quotient of functions, of the superposition, of the inverse function – all items with examples).**
19. **Equation of the line which is tangent to a differentiable function.**
20. **Rolle and Lagrange theorems on a mean value in the differential calculus.**
21. **Fermat theorems on extrema and on the monotonicity.**
22. **Differential investigations on the convexity of a function.**
23. **De l'Hopital rules. Finding the direction coefficient of an asymptotic line.**
24. **AVSPAMECI (arguments,values,symmetry,periodicity,asymptotics, monotonicity and extrema, convexity and inflections) - a schema to recognize the shape of a curve $y = f(x)$.**
25. **Antiderivative and an indefinite integral – definitions, geometric interpretations and examples.**
26. **Riemann integral and its geometrical interpretation. Newton-Leibniz fundamental theorem in the calculus.**
27. **The Gauss/bell curve $y = \exp(-x^2)/(2\pi)^{1/2}$ and a normal distribution.**
28. **Newton cooling problem, its description via an ordinary differential equation of the 1st order (ODE1) and its solution.**
29. **Complex numbers. Euler formula: $\exp z = e^r\cdot(\cos u+i\cdot\sin u)$, where $z=r+i\cdot u$, and $r, u \in R$.**
30. **Linear ordinary differential equation of the 2nd order (ODE2) and its solution (when its characteristic equation has two distinct real zeros, a double zero, and complex solutions).**